Narrow transmittance peaks in a multilayered microsphere with a quasiperiodic left-handed stack

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We studied the photon spectrum in coated microspheres with alternating quasiperiodic layers having left-handed (LH) materials included. It is found that the band gap (spectral zone of nearly zero transmittancy) in such a system is strongly enhanced. At an increase of the quasiperiodicity parameter γ, the boundaries of the spectral band gaps acquire the intended shape. When γ exceeds the inverse golden mean value, a structure with extremely narrow separated resonant peaks with nearly complete transmittance arises. At increased γ, such resonances migrate to the center of the band gap. This effect allows creating optical filters with extremely narrow passbands.

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1. Introduction

1.1. Microspheres

Photonic crystals are artificial materials with a periodic modulation in the dielectric constant which can create a range of forbidden frequencies known as a photonic band gap [1]. Photons with frequencies within the band gap cannot propagate through the medium. This unique feature can dramatically alter the properties of light, enabling e.g., control of spontaneous emission in quantum devices and light manipulation for photonic information technology [2]. It is well known that except for the whispering gallery mode (WGM) regime, a bare dielectric sphere has a complex spectrum of electromagnetic low quality (Q factor) eigenoscillations because of the energy leakage into the outer space [3]. The case of the compound structure, when the dielectric sphere is coated by an alternative stack, is much richer. The Q factor of optical oscillations has a large value in the frequency regions of weak transmittance, and beyond these regions, Q remains small [4–6]. This gives rise to a large variety of optical properties of microspheres coated with multilayer stacks. Such a system can serve as a spherical symmetric photonic band gap structure, which possesses strong selective transmittance properties [7,8], with the incorporated nanometer-sized photon emitters. These possibilities essentially allow the expansion of the operational properties of microspheres with attractive artificial light sources for advanced optical technologies. The energy confinement into small volumes has applications in many fields such as photonics and quantum electronics. In this sense, the microstructures that have the greatest ability to store energy for long periods of time are the dielectric spherical resonators. Recently, it has been feasible to construct such a microsphere accurately, and the parameters may be precisely controlled and measured [9]. New materials are used to extend the optical properties of layered microspheres. Authors [10] have produced optically active nanostructures comprised of polymer multilayer microspheres with four alternating layers with different refractive indices surrounding semiconductor quantum dots.

1.2. Metamaterials

Recently, there have been many studies about metamaterials that have a negative refractive index n. These left-handed (LH) materials, theoretically discussed by Veselago [11], have negative electrical permittivity and magnetic permeability simultaneously. Such materials, consisting of split ring resonators (SRRs) and continuous wires, were first introduced by Pendry [12,13], who suggested that they can also act as effective lenses [14]. Negative-index metamaterials that exhibit unique refractive properties in IR and a visible spectrum are currently a focus of research in optoelectronics, see e.g. [15,16]; and references therein. Various experimental and theoretical investigations of such structures were recently developed. In a metamaterial with the magnitude of the real part of the index comparable to the imaginary part, over 40% transmission can be achieved in the negative-index region by structural adjustment [17]. Most of the unusual properties and applications of metamaterials in optical communications and data processing require modulation of the effective negative refractive index that can normally be achieved by
modulating the optical properties of such structures. Using the pump/probe method, a pump-induced change of the effective negative refractive index of the metamaterials composition Ag/Si/Ag heterostructure [18] has already been observed. The waveguide properties of such structures that depend on the effective permittivity and the refractive index are studied theoretically with details for various models. It was shown [19] that layered heterostructures combining ordinary and negative refractive index materials display a new type of photonic band gap. The transmission and reflection data obtained through transfer matrix calculations on metamaterials [20] of finite lengths was applied to determine effective permittivity $\varepsilon$ and permeability $\mu$. Using the effective-field theory [21], the nanofabricated media with magnetic response in the visible spectrum was analyzed. The analysis of refractive index properties of optical metamaterials as a function of real and imaginary parts of dielectric permittivity and magnetic permeability [22] demonstrated specific interplay between the resonant response of constituents of metamaterials that allows efficient dispersion management. The use of material of one-dimensional plane, periodical structures, including LH layers, allows considerable widening of the band gap of layered structures [23]. Besides, a one-dimensional periodic structure containing the layers of transparent left-handed metamaterial can trap light in three-dimensional space due to the existence of a complete band gap [24]. The natural extension of such investigation is an analysis of a spherical multilayered system with an alternating stack having LH layers included. In studying such an effect in multilayered microspheres, the following question emerges: How does such an effect change at a deviation from a strict periodicity case? In [25,26] optical properties of a layered microsphere with a dielectric stack, in which the optical layers are constructed following the Fibonacci sequence, were investigated. Such a quasiperiodic structure serves as an intermediate case between a strictly periodic and cleanly casual sequence of alternative layers in a stack. It was found that when the number of layers (Fibonacci order) increases, the structure of the spectrum acquires a fractal form. The width of the resonant peaks in the frequency spectrum becomes extremely narrow for a spherical stack of high Fibonacci order. The following idea emerges: to combine the conventional and LH materials (which lead to widening of the band gap) in the quasiperiodic (Fibonacci) spherical stack. As a result of optical re-reflections in such a system, the initial spectral band edges will be progressively reproduced in a much smaller frequency scale (the self-similarity property) that finally leads to the formation of narrow transmittance peaks in the band gap.

In this paper, we study the properties of spherical structures containing alternating conventional and left-handed (LH) layers that are transparent and can bend light in opposite directions. We have found a substantially enhanced gap bandwidth in such structures. When the ratio of the layer width in two-layer blocks in a stack (quasiperiodicity parameter) is close to the inverse golden mean value, a narrow, well-separated resonant peak with nearly complete transmittance arises. As far as the authors are aware, the optical fields in a microsphere with such a compound quasiperiodic spherical stack have still been poorly considered, though it is a logical extension of previous work in this area. This paper is organized as follows: In Section 2, we formulate a transfer matrix approach for multilayered microspheres with a quasiperiodic spherical stack. In Section 3, we present our numerical studies of the features of the transmittance spectrum depending on the quasiperiodicity parameter, spherical quantum numbers and the Fibonacci order. We observed the formation of extremely narrow peaks of high transmittance in the gap band when the quasiperiodicity parameter $\gamma$ closely exceeds the inverse golden mean value. Such an effect arises in quasiperiodic layers and is absent in strictly periodic cases. The influence of a weak random deviation of spherical layers thickness is also discussed. In Section 4 we discuss our study, while in the last section, we summarize our results.

2. Basic equation

A 1D quasiperiodic (QP) spherical stack (see Fig. 1), where a Fibonacci sequence is considered, can be constructed following a simple procedure. Let us consider two neighboring two-layer segments, long and short, denoted respectively, by $L$ and $S$. If we place them one by one onto the surface of a microsphere, we obtain a sequence: $LS$. In order to obtain a QP sequence, these elements are transformed according to Fibonacci rules as follows: $L$ is replaced by $LS$; $S$ is replaced by $L: L \rightarrow LS; S \rightarrow L$. As a result, we obtain a new sequence: $LSL$. Iteratively applying this rule, we obtain, in the next iteration, a sequence with a five-element stack $LSLS$, and so on. One can control the properties of such a QP stack by the use of some control parameter (see factor $\gamma$ later in Section Numerical results). For the stack with $N$-elements, where $N \gg 1$, the ratio of numbers of long to short elements is the golden mean value, $\tau_0 = (1 + \sqrt{5})/2 \approx 1.618$.

Systems with 1D Fibonacci stacks (where the translation symmetry takes place) were investigated in a number of works [27–32] by means of a renormalization-group (RG) method associated with the transfer matrix relation $M_{j+1} = M_j M_{j-1}$. Nevertheless, these results cannot be applied directly to a spherical stack for various reasons: i) In a spherical case, the transfer matrices $M_j$ have the determinant depending on number $j$ of a layer as $\det(M_j) = (r_{j+1}/r_j)^2 > 1$ [6] (is not unimodular), where $r_{j+1}$ and $r_j$ are radii of the external and internal boundaries of the layer, respectively. Such a structure of the transfer matrix is physically caused by preservation of the energy flux in a radial direction in a solid spherical angle. ii) In a spherical stack, the transfer matrix $M_j$ is quite involved since it is written through the complex Hankel Functions. Physically, it is due to the preferential role of the pole (center) of a microsphere that breaks translational symmetry in such a system. Furthermore, in a spherical geometry, the transmittance coefficient $T$ depends on the spherical quantum number $m$ (angular momentum). For large $m \gg 1$ that leads to a whispering gallery mode (WGM) regime with practically zero transmittance in a frequency range. In order to study the optical properties of a spherical Fibonacci stack, let us first formulate the transfer matrix method exploited in spherical multilayered geometry. Various approaches were proposed [6,33–36] (see [37,38] and references therein). Here we follow [33]. In a multilayered microsphere, the set of the Maxwell equations is

$$\nabla \times \overrightarrow{H} = i \omega \mu_0 c (\varepsilon(\omega)) \overrightarrow{E}; \nabla \times \overrightarrow{E} = -i \omega \overrightarrow{B},$$

(1)

where $\overrightarrow{E}$ and $\overrightarrow{B}$ are electric and magnetic fields, and $\varepsilon(\omega)$ is a dielectric permittivity of a layer. We use the complex exponential multiplier in the form exp$i(\omega t)$. Eq. (1) in the spherical coordinate frame ($\rho, \theta, \phi$) usually reduces to the Helmholtz equation for a scalar function called the Debye potential $\Pi(\rho, \theta, \phi)$ [39,40]. To avoid repetition we concentrate on the TE wave case; TM wave case can be studied in the same way. The equation for the radial part of the Debye potential $\Pi = \Pi(r)$ in a layer can be readily obtained from (1) and is given by

$$\frac{d^2 \Pi}{dr^2} + \left[\varepsilon(\omega) k_0^2 - \frac{l(l + 1)}{r^2}\right] \Pi = 0,$$

(2)

where $k_0 = \omega/c$. Eq. (2) is easily solved in terms of spherical Hankel functions. In each layer of the stack, we use the next matrix presentation for the fields

$$\overrightarrow{u} = \begin{bmatrix} H_l \varepsilon_0 \\ E_l \end{bmatrix} = D \begin{bmatrix} a \\ b \end{bmatrix} = D \overrightarrow{q}, \quad \overrightarrow{q} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

(3)
where \( a \) and \( b \) are arbitrary constants, and matrix \( D = D(y) \) is given by

\[
D = \begin{bmatrix}
    ip_m^{1/2}(y)e^{iy} & ip_m^{1/2}(y)e^{-iy} \\
    G_m^{1/2}(y)e^{iy} & G_m^{1/2}(y)e^{-iy}
\end{bmatrix}.
\]  (4)

Here, \( n = n(\omega) = \sqrt{\varepsilon(\omega)} \) is the refractive index of a particular layer (that can be positive or negative); \( P_m^{1/2}(y) \) is the rational part of derivative of Hankel spherical functions \( \partial/\partial y \), \( G_m^{1/2}(y) \) is the rational part of Hankel spherical functions \( P_m^{1/2}(y) \), \( m \) is the number of a spherical harmonic, \( y = \omega r / c \). The recursive relations for calculations of \( P_m^{1/2}(y) \) and \( G_m^{1/2}(y) \) are given in [34]. Below, we provide the matrix \( D \) as well as vectors \( q \) and \( u \) by indices accordingly to the number of layers in the stack. As the vector \( q \) in each layer is constant, for any two points \( r_1 \) and \( r_2 \) of a \( k \)-layer we obtain from Eq. (3) the following:

\[
\vec{q}_k = D_k^{-1}(r_1) \cdot \vec{u}_k(r_1) = D_k^{-1}(r_2) \cdot \vec{u}_k(r_2).
\]  (5)

To start the calculation, we assume that the points \( r_1 \) and \( r_2 \) belong to the boundaries of layers; \( r_2 = r_1 + d, \) \( d \) is the thickness of the layer. At the boundary between layers, \( k \) and \( k+1 \), the continuity of fields gives \( \vec{u}_k(r_2) = \vec{u}_{k+1}(r_2) \). With the latter Eq. (5) can be rewritten as

\[
\vec{u}_k(r_1) = D_k(r_1) \cdot D_k^{-1}(r_2) \cdot \vec{u}_k(r_2) \equiv M_k \cdot \vec{u}_k + 1(r_2) \cdot \vec{u}_k + 1(r_2).
\]  (6)

This relation can be extended as follows. Let us start from the inside layer of the stack with the number \( k = 1 \). Then from Eq. (6) we have

\[
\vec{u}_1 = \vec{u}_1(r_1) = M_1 \cdot \vec{u}_1 = M_2 \cdot \vec{u}_2 = ... = M_{N-1} \cdot \vec{u}_{N-1} = \vec{u}_N;
\]

\[
\equiv \vec{M}^{T} \cdot \vec{u}_N = \vec{M}^{T} \cdot \vec{u}_N(r_N).
\]  (7)

where

\[
M = \prod_{k=1}^{N-1} M_k
\]  (8)

is the transfer matrix between inner and outer layers in the spherical stack. Now, we exploit such a technique for a quasiperiodic (Fibonacci) spherical stack with LH metamaterials included. We use a block of two contacting neighbor layers as an elementary cell. All cells differ only by the width of the second layer \( \lambda \) and the width of the first layer in block \( k \) is fixed. There are two types of cells in the stack: \( L = (l_1, l_2, \gamma) \) is a long block with two \( \lambda/4 \) layers (\( \gamma = 1 \)), while \( S = (l_1, l_2, \gamma) \) is a short block (\( \gamma \leq 1 \)). Here, \( \gamma \) is the ratio of the second layers width in \( S \) and \( L \) blocks: \( \gamma = l_2/S(l_2/L) \), so that \( 0 \leq \gamma \leq 1 \). To avoid an ambiguity between one-layer and two-layer transfer matrices, we denote the two-layer transfer matrix as \( K_2 = M_2 \cdots M_2 \), where \( F \) is the number of two-layer blocks and \( N = 2F \) is the number of layers in the stack. To reconstruct the entire stack, we use the “initial” conditions: \( K_0 = (S), K_1 = (L) \). The Fibonacci order is defined by numbers \( F_0 \) with \( F_0 = 0, F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, F_7 = 13, F_8 = 21, ... F_{j+1} = F_j + F_{j-1} \), etc. Now we reconstruct the spherical QP stack as follows. First, on the surface of a microsphere, there is one stack \( L(1) \) (first) block is placed, so \( K_1 = (L) \). In the second step, block \( S(2) \) is attached to the stack on the right (external side) and \( K_2 = (L(1), S(2)) \). From the latter, we observe that a spherical stack cannot be constructed by application of the standard recursion: \( K_j = (K_j K_1) = (LSL) \). Such a recursion is applicable only for a system where the translation symmetry holds, and the transfer matrix depends only on the thickness of a block, but also on the distance to the microsphere center. So in the construction of spherical stack, one has to explicitly specify the number \( j \) of blocks in the stack. Blocks \( L(i) \) and \( L(j) \).
are not equivalent to \( i \neq j \). For derivation of a general formula for a spherical Fibonacci stack, we write down the following equations:

\[
K_1(1) = \{L(1)\}; \\
K_2(2) = \{L(1)S(2)\}; \\
K_3(3) = \{L(1)S(2), L(3)\} = K_2(2)K_1(1); \\
K_4(4) = \{L(1)S(2)L(3), L(4)S(5)\} = K_3(3)K_2(2)(F_4+1),
\]

where \( F_n \) is Fibonacci order. Further, we obtain

\[
K_5(5) = K_4(4)K_3(3)K_2(2)K_1(1)K_{F_6},
\]

where \( F_n \) is Fibonacci order. Further, we obtain

\[
K_{j+1}(j+1) = K_j(j)K_{j-1}(F_j+2),
\]

and cannot be reduced to a plane case. Therefore, the properties of the spherical Fibonacci stack differ from the plane stack. By indicating

\[
\text{Fig. 2. Comparison of the transmission coefficient } T \text{ spectrum as a function of frequency } f/f_0 \text{ (with } f_0 = 171.5 \text{ THz) and spherical number } m = 1 \text{ for LH stack (with LH material LHM) and conventional stack. The Fibonacci order is } F_8 = 21 \text{ (21 two-layer blocks) and quasiperiodicity parameter } \gamma = 0.66. \text{ The structures of the two-layer block are: (a) } \text{SiO}_2 + \text{LHM}; \text{ (b) } \text{LHM } + \text{SiO}_2; \text{ (c) } \text{SiO}_2 \text{ Si; and (d) } \text{Si, SiO}_2. \text{ In (a) dash line indicates the position of the boundary gaps for } \gamma = 1 \text{ (strictly periodic } } \lambda/4 \text{ stack). See details in text.}
\]

\[
\text{Fig. 3. Dependence of the transmission coefficient } T \text{ on the frequency } f/f_0 \text{ with } f_0 = 171.5 \text{ THz for different values of the quasiperiodicity parameter } \gamma, \text{ and the Fibonacci order } F_8 = 21 \text{ (21 two-layer blocks). See details in text.}
\]
number $F_j$ (as well as the materials and thickness of layers), the structure of a spherical stack is defined completely. In order to explore such a Fibonacci stack, we will exploit the transfer matrix approach that provides a procedure to analyze a continuous transition from a periodic to a QP (Fibonacci sequence) spherical stack. Eq. (8) for transfer matrix cannot be written in a compact form; thus it is too complicated a problem to explore it analytically. Therefore, below we study the transmittance spectrum for quasiperiodic layers numerically (more details about the use of SPM).

**Fig. 4.** Transmission coefficient $T$ in the frequency range $f / f_0 = 1.75$ to 4.25 with $f_0 = 171.5$ THz, where quasiperiodicity parameter is $\gamma = 0.66$ for different values of the spherical number $m$: (a) $m = 1$, (b) $m = 6$, (c) $m = 11$, (d) $m = 13$, at the Fibonacci order $F_8 = 21$ (21 two-layer blocks).

**Fig. 5.** Details of the transmission coefficient $T$ in the frequency range $f / f_0$ from 1.75 to 4.25 (with $f_0 = 171.5$ THz), for spherical number $m = 1$ for different values of quasiperiodicity parameter $\gamma$: (a) $\gamma = 0.66$, (b) $\gamma = 0.67$, (c) $\gamma = 0.68$ and (d) $\gamma = 0.69$. The Fibonacci order is $F_8 = 21$ (21 two-layer blocks).
of the transfer matrix method for the spherical geometry one can find in [6,26,37]).

3. Numerical results

Our numerical results are shown in Figs. 2–9.

First we compare the spectral behavior of the transmittance coefficient $T$ for cases the spherical stack having left-handed materials (LHM) included and the spherical stack with conventional materials. We apply the transfer matrix technique developed in the previous section that is independent on the material type. The frequency spectrums for both cases are depicted in Fig. 2. In Fig. 2(a) and (b) the spectrum of the LH stack is

Fig. 6. Transmission coefficient $T$ for spherical numbers $m = 1$, and the quasiperiodicity parameter $\gamma = 0.66$ for different values of the Fibonacci order $F_n$ (a) $F_3 = 3$ (3 two-layer blocks), (b) $F_5 = 5$ (5 two-layer blocks), (c) $F_8 = 8$ (8 two-layer blocks), (d) $F_{13} = 13$ (13 two-layer blocks).

Fig. 7. Transmission coefficient $T$ for spherical numbers $m = 1$, and the quasiperiodicity parameter $\gamma = 0.66$ for different values of the Fibonacci order $F_n$ (a) $F_8 = 21$ (21 two-layer blocks), (b) $F_9 = 34$ (34 two-layer blocks), (c) $F_{10} = 55$ (55 two-layer blocks), (d) $F_{11} = 89$ (89 two-layer blocks).
shown, while Fig. 2(c) and (d) demonstrates the spectrum of the spherical stack with conventional materials. From Fig. 2, we observe that the width of the band gap for LH stack is considerably wider in regard to the conventional material stack. We notice that similar behavior was pointed out in Ref. [23] for a plane one-dimensional periodic LH stack. Fig. 2(a), (b), (c), and (d) shows that this property does not change significantly for the structures of two-layer blocks in the quasiperiodic stack. Some difference between Fig. 2(a) and (b) (and also Fig. 2(c) and (d)) reflects the break of the radial-translating symmetry in such a system. Since the properties of a quasiperiodic conventional spherical stack were studied with details in Ref. [26], further we will consider the transmitting properties of the spherical quasiperiodic LH stack. The following parameters have been used in our calculations: the geometry of system is $A_{L}B_{C}...S_{B}C_{S}...D$, where letters $A$, $B$, $C$, and $D$ indicate the materials in the spherical stack, respectively; see Fig. 1. The inside microsphere has a refraction index of $n_{4}=1.5+10^{-4}i$ (glass, radius 1000 nm). The distinct two-layered blocks $(L_{B}C_{S})$ and $(S_{B}C_{L})$ are stacked according to the Fibonacci generation rule. For $L$ and $S$ blocks we use the notation $L=(B,C,1)$ and $S=(B,C,\gamma)$, where $\gamma$ is the ratio of both thicknesses: $\gamma = (C_{S}/C_{L}) \leq 1$. Refraction indices of the layers in the blocks are: (as LH metamaterial (LHM) we use the medium with the following typical parameters) $n_{C}=-3.58+9 \cdot 10^{-4}i$ (LHM), $n_{B}=1.46+10^{-3}i$ (SiO$_2$) [41] and $n_{D}=1$ (D, surrounding space). In Fig. 3, the frequency dependence of transmittance coefficient $T$ in the band gap zone is shown at various values of the quasiperiodicity parameter and $F_{8}$ Fibonacci order, with 42 layers (21 two-layer blocks), where the width is $8.65 \mu m$. We observe that the boundaries of the zones are located in $f/f_{0}=2n$, $n=1,2,3...$ [37]. The structure of the gap zone at small values of parameter quasiperiodicity $\gamma$ close 0.66 does not depend on frequency. However, the situation changes when $\gamma$ approaches 0.618 (the inverse golden mean value $1/\Gamma_{0}$, $\Gamma_{0}=1.618$). The compact structure of peaks with high values of transmittance coefficient starts to split from the boundary zone at $f/f_{0}=2n$, where $n=2$. With an increase $\gamma$, this structure acquires the form of a triplet and migrates throughout the gap to the opposite boundary with $n=1$. At $\gamma$ close 1, the triplet merges into the nearest boundary with $n=1,3$. In Fig. 3 we also observe that similar dynamics occur in the next zone with $n=3,4$; however, with the peaks of smaller amplitudes.

![Fig. 8. Details of main peaks in the triplet as function of the photons wavelength (nm). The spherical number is $m=1$, quasiperiodicity parameter $\gamma=0.66$. (a) Details of the triplet in Fig. 7(a) with Fibonacci order $F_{8}=21$ (21 two-layer blocks); (b) details of the triplet in Fig. 7(b) for the Fibonacci order $F_{9}=34$ (34 two-layer blocks). The insets in (a) and (b) show the details of highest peaks. We observe that FWHM (full width at half maximum) $d$ of these peaks are: (a) $d=0.4$ nm, and (b) $d=0.04$ nm, correspondingly.](image)

![Fig. 9. Transmission coefficient $T$ in the frequency range $f/f_{0}$ from 1.75 to 4.25 (with $f_{0}=171.5$ THz) for spherical numbers $m=1$, Fibonacci order $F_{7}=13$ (13 two-layer blocks), and the quasiperiodicity parameter $\gamma=0.66$ for different values of the random deviation $r$ in widths of the layers: (a) 1%, (b) 10%, (c) 15%, (d) 25%. Insets in (a) and (d) show the details of highest peaks in corresponding frequency ranges.](image)
As the field distribution in microspheres does not possess the radial-translating symmetry, such a behavior practically is not repeated in higher order zones. Fig. 3 clearly shows that the formation of transmittance peaks in the center of the gap zone is due to the quasiperiodic property of the stack and is absent in a periodic case, when \( \gamma = 1 \). Further, we investigated this effect at various \( \gamma \), with spherical quantum numbers \( m \), and the number of layers in a spherical stack \( F_n \) (Fibonacci order).

Fig. 4 shows the behavior of \( T \)-peaks at different values of the spherical number \( m \). We observed from Fig. 4(c) that from \( m = 11 \) the amplitude of peaks already decreases notably. Dependence on the spherical number \( m \) is rather instructive. At a large \( m \), there is a transition to the WGM (whispering gallery mode) regime when the photon field is exponentially localized near the surface of the microsphere. In this regime, the field does not “feel” the details of the spherical stack structure. Thus, with the increase \( m \) the amplitude of the peaks should decrease rapidly. Such behavior has been registered clearly in our numerical experiments.

Fig. 5 shows the \( T \) spectrum for \( \gamma = 0.66 \) to 0.69, when the triplet is already well separated from the boundary. We observe from Fig. 5 that with increment of \( \gamma \), the triplet migrates to the opposite boundary while in the boundaries there is practically no change. The amplitude of the peaks is rather high and reaches \( T = 0.99 \). At a further increase of \( \gamma \) (see Fig. 5(d)), the amplitudes of the structure decrease and then merges to the close boundary.

Now, we explore the details of peaks for various numbers of quasiperiodic layers in the spherical stack (the Fibonacci order). Our purpose is to find the optimal number of quasiperiodic layers in a stack when the maximal value of transmittance \( T \approx 1 \) can be reached. We observe from Fig. 5 that the spectral bandwidth of the transmittance peaks is narrow with respect to the bandwidth of the transmittance \( T \) at the area of the gap zone boundaries (GZB) at \( f/f_0 = 2, 4, \ldots \). Such peaks could serve as a bandwidth filter with very narrow bands, however the following question emerges: Is there a way in which one could increment the amplitude of \( T \) close to 1? We answer this by investigating the change of the Fibonacci order of two-layer blocks in the stack. Fig. 6 shows the change of the shape of the peaks at an increase of the Fibonacci order \( F_0 \) at \( m = 1 \) and \( \gamma = 0.66 \).

One can see that structure of spectrum shown in Fig. 6(c) and (d) is similar to the structure in Fig. 6(a) and (b), but within of much smaller frequency range (near \( f/f_0 \approx 3 \)). This is known signature of self-similarity of quasiperiodic systems due to progressive recursive reconstruction of the stack matrices, see Eq. (9). At small Fibonacci order the spectrum shown in Fig. 6(a) is modified rapidly with increase of \( F_0 \). We observe from Fig. 6(d) that already at Fibonacci order \( F_2 \) the resonance linewidth \( T = 0.98 \) is generated that can be promising for optical filters. Thus it is interesting to study the structure of resonant peaks for larger \( F_0 \) values. Results of calculations depicted in Fig. 7(a)–(d) show that already from order \( F_0 \) all resonances are narrow increasing. It is worth noting that linewidth in Fig. 7 is so small that better resolution in vicinity of peaks is required.

More details of the main peaks in the triplets with better resolution as function of the photons wavelength (nm) are shown in Fig. 8 for Fibonacci orders \( F_0 = 21 \) (21 two-layer blocks) and \( F_0 = 34 \) (34 two-layer blocks), for spherical number \( m = 1 \) and quasiperiodicity parameter \( \gamma = 0.66 \). That gives us possibility to compare our results (actual numbers for linewidth) with data known from literature (see Section 4). The insets in Fig. 8(a) and (b) show the details of highest peaks. We observe that FWHM (full width at half maximum) \( d \) of these peaks are: \( (a) \) \( d = 0.4 \) nm, and \( (b) \) \( d = 0.04 \) nm, correspondingly. For higher values \( F_0 \), the width of the peaks acquires even smaller values \( d \) that allows creating very selective stop-band filters with high transmittance \( T \).

Although above we paid attention mainly to the pure quasiperiodic case, we finally study what happens if the width of the layers in the LH Fibonacci stack has some random deviations (roughness). In this case, the (quasi)periodicity in the system due to re-reflections of the waves is broken. Results are shown in Fig. 9 for different levels of the random deviation \( r \). We observe that even for not small value of the roughness factors the spectrum still contains the peaks with high transmittance.

4. Discussion

The behavior of such a system generally can be explained starting from a simple plane periodic layered medium consisting of two different materials with widths \( a \) and impedances \( n_i = (\mu_i/\varepsilon_i)^{1/2} \). The first layer is dielectric, while second layer may be LH material with \( \varepsilon < 0 \) and \( \mu > 0 \). The dispersion relation with \( \omega = 2\pi f \) versus \( \delta(a) \) for 1D plane geometry can be written as (see e.g. [23], [43])

\[
\cos(\delta) = \cos(\phi) \cos(\alpha) - q \sin(\phi) \sin(\alpha p),
\]

with \( p = \omega_0 a_1/c, \alpha = n_2 a_2/n_1 a_1, q = (1/2)(\eta_1/\eta_2 + \eta_2/\eta_1) \geq 1, \alpha = \pm 1, \) where \( \sigma = -1 \) for LH material, and \( n_i \) are the refractive indices. Eq. (10) is solved for \( a \) to determine the forbidden bands. Here the negative sign of the refractive index is shown explicitly (by means of parameter \( \sigma \)), so in Eq. (10) both refractive indices \( n_1, n_2 \) are all positive. For quarter-wave stack \( \alpha = 1 \) and \( \sigma = 1 \) (conventional layer) Eq. (10) is reduced as follows

\[
\cos(\delta) = \cos^2(p) - q \sin^2(p),
\]

that has well-known solution to determine the properties of the band gaps for a conventional \( \lambda/4 \) dielectric plane stack, see e.g. Chap. 6 in [42]. However for \( \sigma = -1 \) (LH layer) in contrast to \( \sigma = 1 \) case the equation

\[
\cos(\delta) = \cos^2(p) + q \sin^2(p),
\]

has a real solution for \( a \) only for \( p = \pi n (n = 1, 2, \ldots) \) independently of material impedances (parameter \( q \)). This means that in Eq. (11) the transmittance bands shrinks to zero [23]. For a layered microsphere with the periodic stack the corresponding spectrum is calculated and depicted in Fig. 2(a) (dash line) with narrow peaks of transmittance for band edges \( f/f_0 = 2, 4 \) and \( \gamma = 1 \) (strictly periodic stack). However we observe from Fig. 2 that for quasiperiodic stack the spectrum splits and acquires fairly indented form already for \( \gamma = 0.66 \). Fig. 6 shows that if the number of two-layers blocks in the stack (order Fibonacci \( F_0 \)) increases, then even smooth parts of the spectrum (the band edges at \( f/f_0 = 2, 4 \)) acquire a fractal shape, see Fig. 6(d) and further Fig. 7. Such a behavior can be interpreted as a signature of self-similarity that is a manifestation of fractal systems [31]. We observe that the spectrum structure in Fig. 6(c) and (d) in the range closely to \( f/f_0 = 3 \) is rather similar to the spectrum depicted in Fig. 6(a), but for larger interval \( f/f_0 \) from 1.5 to 4.5. Some discrepancies in Fig. 6(a) and (d) are to be expected (see more details in [26]) due to the dependence of the transfer matrix \( M \) in Eq. (8) on the number of a layer in the stack and that the determinant of the transition matrix depends explicitly on number \( j \) of a layer as \( \det(M_j) = (r_{j+1}/r_j)^2 > 1 \) (is not unimodular, as is requires the theory of a plan geometry, see [30] and references therein). Nevertheless in agree with [26–31] one can see that when the number of layers in the stack (Fibonacci order) increases, the structure of the transmittance spectrum acquires fractal form with narrow resonances (peaks) and quasiband gaps due to re-reflection of light from total quasiperiodic structure. It is worth to note that calculating of the spectrum details with fine resolution for \( F_0 > 1 \) requires much more detailed datapoints and generally requires deeper investigation. Quantitatively such spectral structures in microspheres can be described by the fractal dimension, for further details see [26] and references therein.

As it was mentioned above from the inset in Fig. 8, we can observe that the bandwidth of the transmittance peaks is much smaller than the width of the band gap. Thus it is interesting to make use of the extremely narrow resonate peaks in the bandgap region to create selective optical
filters with extremely narrow passbands for studied spherical quasiperiodic structure. It is worth noting that the narrow-band filters with FWHM 0.457 nm and 0.3 nm correspondingly, were recently realized in Refs. [44,45]. In order to study such possibilities for spherical quasiperiodic structures we have investigated the spectral width of narrow resonate peaks in the bandgap region for various Fibonacci orders. Fig. 8 depicts the details of the highest peaks in a triplet structure shown in Fig. 7 as a function of the photon wavelength (nm). The spherical number \( m = 1 \) and the quasiperiodicity parameter \( \gamma = 0.66 \) were used. The insets in Fig. 8(a) and (b) show the FWHM (full width at half maximum) above references. It is worth noting that in our numerical calculations, and even smaller with respect to the spectral widths indicated in the above references. It is worth noting that in our numerical calculations, no significant differences between the TE and TM modes spectra have been observed.

In order to understand the changes in the spectrum shown in Fig. 9 due to random (roughness) factors, we are reminded that the formation of the line structure occurs due to the self-similarity that reproduces the structures of the spectrum in smaller scales. In the case of random deviation, the quasiperiodicity factor becomes weaker; the system migrates from a quasiperiodic state to a random organization. For small random deviations with \( r = 1\% \) the spectrum remains quasiperiodic (see Fig. 9(a)), in this case, small random deviations can be statistically recompensed in the case of a great number of layers. However, as we observe from Fig. 9(b), with the increase \( r = 10\% \) changes occur in the structure of the higher-frequency edge that lead to a restructuring of the central triplet. Fig. 9(c) shows that at a further increase \( r = 15\% \), the shape of the first edge is also deformed substantially. Fig. 9(d) depicts a situation of significant random deviation with \( r = 25\% \); we observe that the quasiperiodic structure of the spectrum is practically destroyed; the high transmittance peak remains only at the low-frequency region close to the first edge. Nevertheless, the height of some narrow peaks may still be high (\( T = 0.99 \), see insets in Fig. 9(d)), but its position is already defined by random factors. We can explain this in the following way. Forming the spectral peaks of transmittance creates a collective effect due to the wave re-reflections from the great number of individual layers. Higher Fibonacci resonances (and self-similarity as well) are more sensitive to the random perturbations of the quasiperiodicity in the system. This can result in the destruction of such high-order structures.

5. Conclusion

We have studied a transmittance spectrum of photons in a layered microsphere with a quasiperiodic subwavelength stack with alternating conventional dielectrics and left-handed material. The spectral evolution of transmittance at the change of the thickness of two-layer blocks, constructed following the Fibonacci sequence, is investigated. We found that at a small quasiperiodicity parameter, the frequency spectrum of the transmittance coefficient is smooth and essentially enhanced in comparison with the case of conventional dielectrics. Such a spectrum is similar to a spectrum of a periodic stack of plane layers with the metamaterials included. However, when the quasiperiodicity parameters reach the critical value equal to 0.618 (inverse gold mean value), a narrow resonant peak of nearly complete transmittance is formed inside the gap zone. At an increase in the Fibonacci order, this peak splits into a triplet; however, the amplitude of the central pulse still remains high and close to 1. At an increase of the spherical quantum number \( m \), the amplitude of peaks decreases rapidly. This is due to that for larger \( m \) a transition to WGM regime occurs; in this situation the field is located practically exponentially closely to the external boundary of a microsphere. In this case, the spatial structure of layers does not render essentially on the spectrum of optical oscillations. Since the field distribution in microspheres does not possess the radial-translating symmetry, such a transmittance structure arises mainly in the first frequency zone. Our main assumption, which was used in this study, is the wideband nondispersive properties of the material forming multilayered spherical structures. More realistic composite materials with desired properties are dispersive and lossy. However, the creation of LH material layers with the required properties has already become possible in the desired frequency range. In this case, the generation of narrow peaks in the center of high reflection spectrum in the gap zone, in principle, allows creating filters with extremely narrow passbands.

References